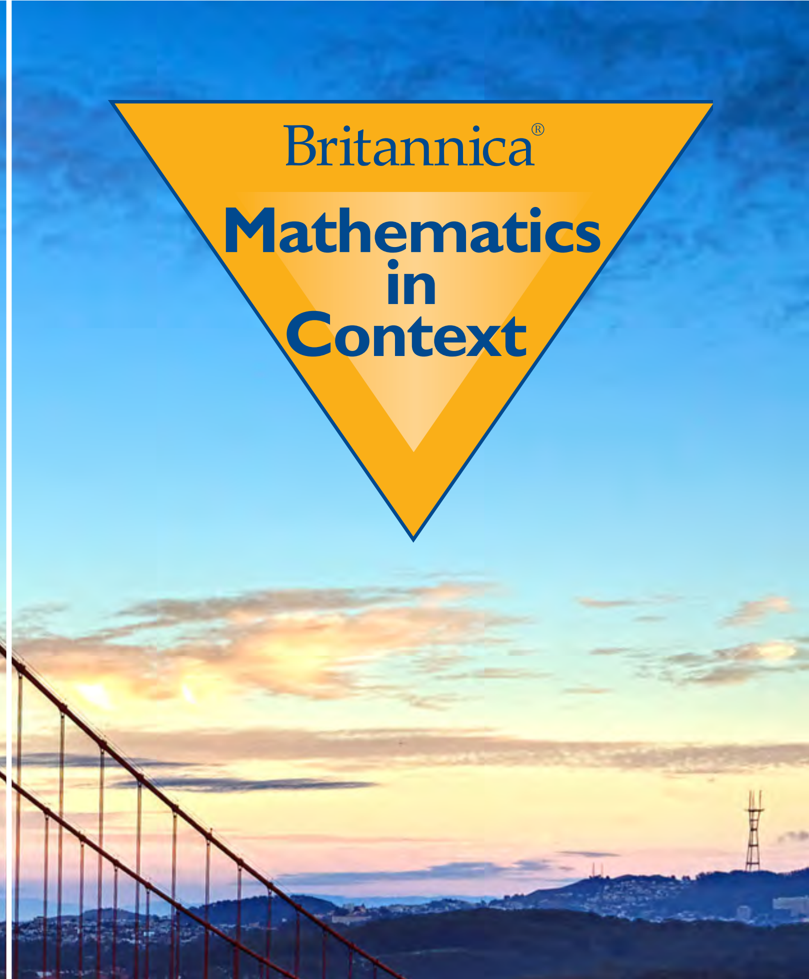
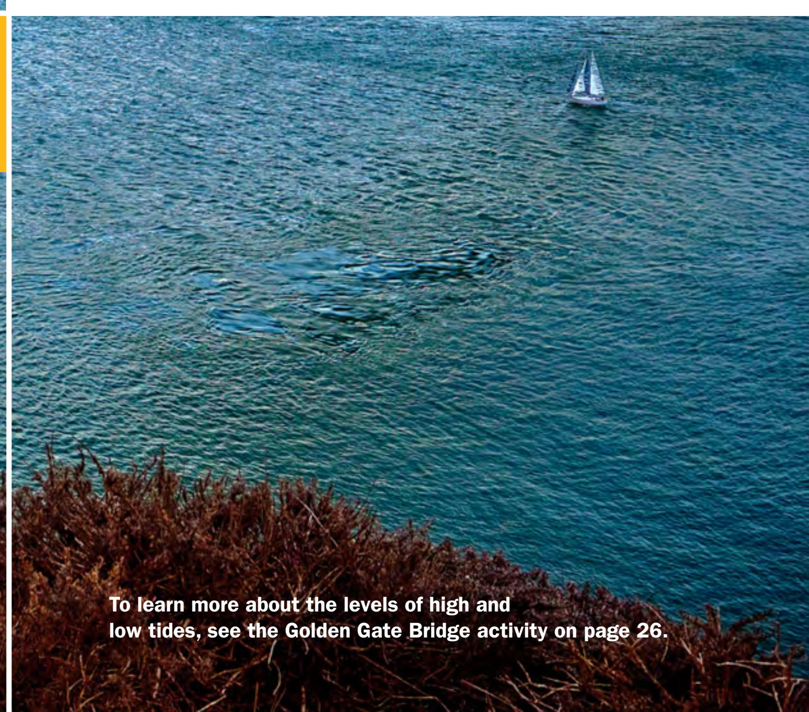


Britannica®

Mathematics in Context

COMMON CORE RESOURCE GUIDE for Middle School

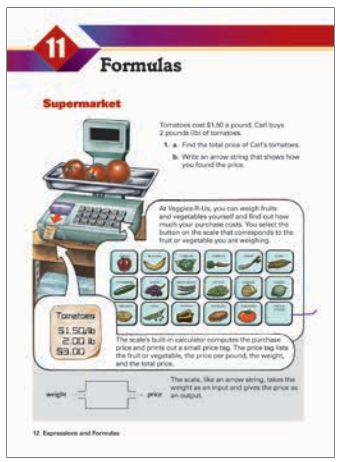


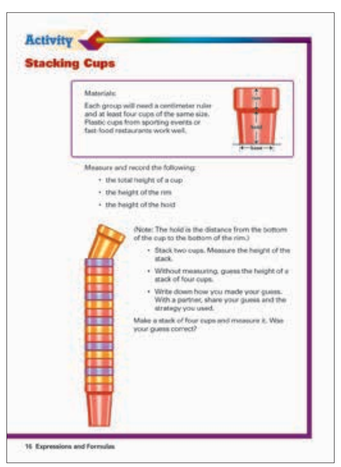
To learn more about the levels of high and low tides, see the Golden Gate Bridge activity on page 26.

CONNECT TO STANDARDS with Mathematics in Context

This guide provides a sampling of real-world math lessons, along with activity sheets, that can be used with students to support the Common Core Math Content Standards in Grades 6 to 8.

These activities and many others appear in Britannica's research-based, NSF-funded *Mathematic in Context* (MiC) resource. The activities use representational models and multiple strategies—pedagogy consistent with the Common Core Math Practice Standards—to help students build key problem-solving and critical thinking skills.





Mathematics in Context (MiC) is a comprehensive mathematics program reaching students of all levels. It motivates reluctant learners from middle school to struggling Algebra 1 students with realistic contexts and multiple strategies while challenging accelerated learners to discover, explore, and understand rich math concepts. The pedagogy is consistent with the Common Core Standards for Mathematical Practice while MiC's content is aligned to the Common Core Content Standards.

Available in multiple formats—print, digital, and interactive—that can be purchased **individually** or as a **whole program**, the MiC program includes:

- 28 individual student titles can be purchased separately—in print, interactive digital, and pdf format
- 28 individual teacher guides that can be purchased separately—in print and pdf format
- *Key to Success* Summer School Program
- Companion Practice Workbooks—one per grade level
- Number Tools and Algebra Tools student workbooks and blackline master teacher guides
- MiC Online
- ExamView® Assessment Suite
- Mathematics in Context Overview
- Manipulative kit
- Professional development

Funded in part by the National Science Foundation

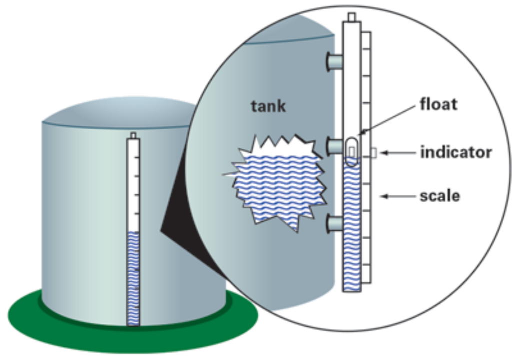
Grade	Lesson	Concepts	Common Core State Standards Mathematics	Standard Description	Pages
6	LESSON #1: *The Bar Model* MiC Unit: *Models You Can Count On*	Calculating and comparing fractions	6.RP.A.1 6.RP.A.3	Understand the concept of a ratio and use ratio language. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g. tape diagrams.	4-11
7	LESSON #2: *Enlarge or Reduce* MiC Unit: *More or Less*	Use of percent as an operator	7.RP.A.2 7.RP.A.2a 7.RP.A.3	Recognize and represent proportional relationships between quantities. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table. Use proportional relationships to solve multistep ratio and percent problems.	12-19
8	LESSON #3: *Systems of Equations* MiC Unit: *Graphing Equations*	Linear equations	8.EE.C.8 8.EE.C.8a 8.EE.C.8b 8.EE.C.8c	Analyze and solve pairs of simultaneous linear equations. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.* Solve real-world and mathematical problems leading to two linear equations in two variables.	20-25
	LESSON #4: *Cyclical Graphs* MiC Unit: *Ups and Downs*	Periodic graphs	8.F.A.3 8.F.B.5	Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	26-31

Notes

You may want to introduce the context of the water tanks with the whole class. If students do not clearly understand how the gauge of a water tank works, you may refer to the gauge of a coffee maker: the gauge on the outside shows the level of the liquid inside.

7 Encourage students to shade the parts as accurately as possible. It may help to suggest that they draw additional lines on the **Student Activity Sheet** to help them see the fractional parts more easily.

Water Tanks



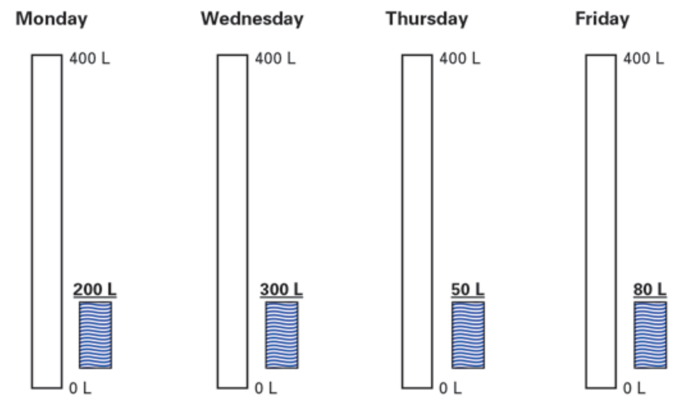
Students use a supply of rainwater, stored in tanks, to water the garden plots.

The largest tank in the garden holds 400 liters (L) of water. However, during a dry spell, it usually has less than 400 L of water.

The outside of the tank has a gauge that shows the level of the water in the tank.

You can use a gauge like a fraction bar.

7. Here is a drawing of the water gauge on four different days.

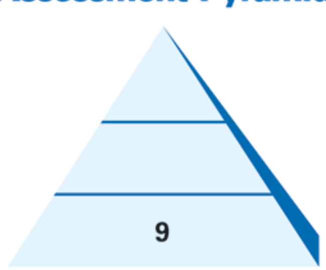


On **Student Activity Sheet 4**, shade each gauge to show the water level indicated for that day.

8. Next to your shading, write the fraction that best describes the water level on each day.

9. Make your own drawing of the gauge on Tuesday. You will need to select the amount of water (in liters) in the tank, shade the part on the gauge, and describe this part with a fraction.

Assessment Pyramid



Use fractions to estimate a quantity relative to a given whole.

Reaching All Learners

Intervention

For struggling students, run additional copies of **Student Activity Sheet 4** so that they can cut the strips out and fold them.

English Language Learners

You may need to read the top of this page aloud to English language learners to ensure understanding of the water tank context.

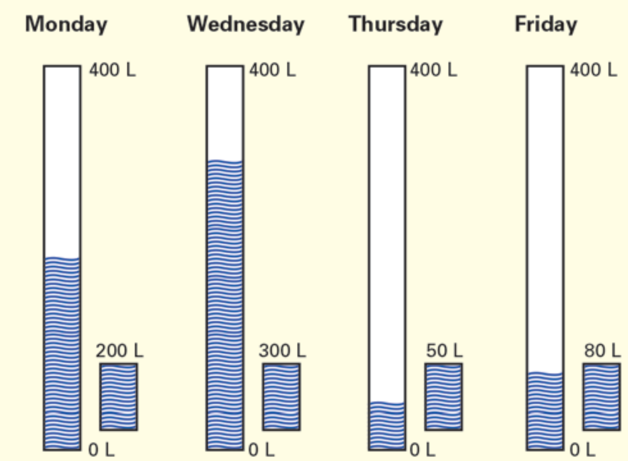
Solutions and Samples

7. and **8.** Monday: Students can divide the gauge in two equal parts, and shade the lower part. Fraction $\frac{1}{2}$

Wednesday: Students can divide the gauge in four equal parts each representing 100 liters, and shade the lower three. Fraction $\frac{3}{4}$.

Thursday: Students can use the division of Wednesday's gauge and divide the upper part in two to get 50 liters. Fraction: $\frac{1}{8}$

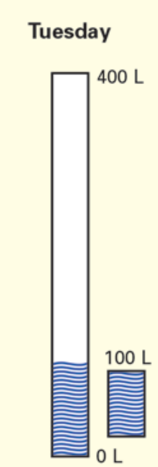
Friday: Students either use the 10 L and 100 L marks they made and estimate where 80 L is, or they use the fact that 80 is the result of 400 divided by 5 and make five equal parts and shade the bottom one. Fraction $\frac{1}{5}$.



8. See answer to problem 7.

9. Answers will vary. Sample answer:

A student may have chosen 100 L and shaded this in the gauge (half of shaded part for Monday). The fraction is $\frac{1}{4}$.



Hints and Comments

Materials

Student Activity Sheet 4 (one per student)

Overview

Students shade the gauge of a water tank to show the water level in the tank on four different days. They use fractions to indicate what part of the tank is filled on each day.

About the Mathematics

The plots, the paper strips, and the water gauges are similar to the fraction bar model. The problems involve the following concepts:

- drawing the correct level on the gauge when a part is given;
- using *(benchmark)* fractions to estimate a quantity relative to a given whole. Benchmark fractions are "easy" fractions, such as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, and $\frac{1}{5}$.

Planning

Students can work on problems 7-9 in small groups. Check students' responses while they are working. If you noticed they are doing well, you may decide just to discuss their work for problem 9. For this problem, you could ask some students to show and explain their work to the class.

Comments About the Solutions

8. There are two different strategies to find the fractions. One strategy is to use the dividing lines on the bar, which is the easiest way. Note that the bars used in this way will give visual support.

The other strategy is to use the part-whole relationships, for example, Thursday: 50 out of 400; 400 divided by 50 is 8, so 50 out of 400 is the same as one out of eight, or $\frac{1}{8}$. Students should be free to choose the strategy they feel most comfortable with.

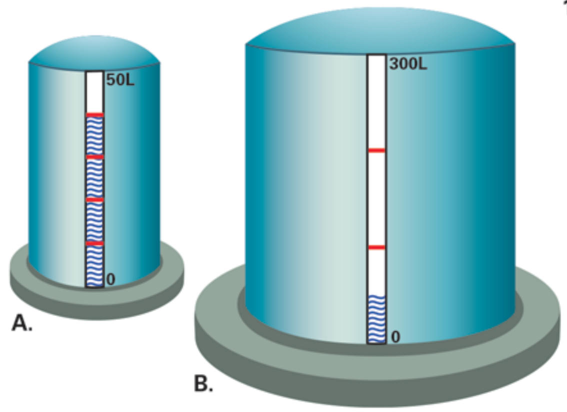
Notes

10 Some students may not understand the concept of capacity. Therefore, you may wish to read the text for problem 10 with the whole class and ask students to explain this concept using their own words.

Discuss students' strategies for problem 10 before they continue with problem 11.

There are different-sized water tanks available at the school garden. By looking at the gauge on a tank, the students can see how much water is inside the tank.

Here are two tanks, one with a water capacity of 50 L and the other 300 L.

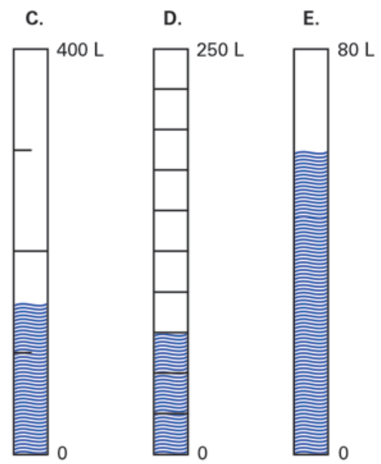


10. a. Explain which of these two water tanks has more water. How did you find out?

b. What fraction of the tank contains water? In your notebook, write the fraction for the shaded area of each gauge.

c. How many liters of water are there in each tank? Write the number of liters in each tank next to the shaded part.

Below are the gauges of three other tanks in the school garden. The maximum capacity of each tank is indicated on top of each gauge.



11. a. What part of each tank is filled? Write each answer as a fraction on **Student Activity Sheet 4** next to the shaded area of the gauge.

b. How many liters of water are in each tank now? Write the number of liters in each tank next to the shaded part.

Reaching All Learners

Intervention

Struggling students may need you to point out the similarities between tanks **C** and **E** to help them solve problem 11.

English Language Learners

Challenge students to find like denominators and use them to compare the fractions in problem 11.

Solutions and Samples

10. a. Answer: Tank **B.**

Sample reasoning:

The gauge of the large tank is divided in three parts. Each part indicates one third of 300 L, which is 100 L. You can see that half of one third is filled, so half of 100 L is in the tank; this is 50 L. That is more than in the smaller tank because the smaller tank can hold 50 L if it is completely filled, but it is not.

b. Tank **A:** $\frac{4}{5}$; Tank **B:** $\frac{1}{6}$, or half of $\frac{1}{3}$;

c. Tank **A:** 40 liters.

Strategies:

Students may have written 10, 20, 30, 40 at the marks, or they may have used the fractions they found in part b.

tank **b:** 50 L

Strategies:

Students may have written 100, 200 at the marks, or they may have used the fractions they found in part b.

11. a. Tank **C:** $\frac{3}{8}$ (students may insert marks between each two marks and thus divide the gauge into eighths

Tank **D:** $\frac{3}{10}$ (students can count the marks and the shaded parts);

Tank **E:** $\frac{3}{4}$ (students may use, for instance, the marks on tank **c**).

b. Tank **C:** 150 L

Sample strategy:

Half of the tank is 200 L, one quarter is 100 L, one half of a quarter is 50 L, so the shaded part means 100 L + 50 L.

Tank **D:** 75 L

Sample strategy: each mark is 25 L.

Tank **E:** 60 L

Sample strategy:

Students may have drawn marks as in tank **C** and written 20, 40, 60.

Hints and Comments

Materials

Student Activity Sheet 4 (one per student)

Overview

Students compare the amount of water in two different tanks with the water level indicated by gauges. The tanks they compare have different capacities. Then they use fractions and liters to describe the part that is filled, indicated by the gauges of three tanks.

About the Mathematics

In problem 10, students make informally relative and absolute comparisons.

Relative: $\frac{4}{5}$ is more than $\frac{1}{6}$.

Absolute: $\frac{4}{5}$ of 50 L is less than $\frac{1}{6}$ of 300 L.

The gauges give students visual support when they start to use a fraction as an operator. Using a fraction as an operator is, for example, $\frac{4}{5}$ of 50 L.

Comments About the Solutions

10. a. Students' reasoning should be based on facts and not on their feelings.

b. The gauge on tank **B** shows a division in three parts, each representing $\frac{1}{3}$, and half of the bottom part is shaded. If students halve the other parts, they may see the relationship that half of $\frac{1}{3}$ is the same as $\frac{1}{6}$.

c. Many students will benefit from the visual support the gauges offer. A strategy that uses the division of the gauge is, for example, for tank **A**: write the number of liters that belongs to each mark (10 L, 20 L, 30 L, and 40 L). In this way, students informally solve $\frac{4}{5}$ of 50 L, which is 40 L.

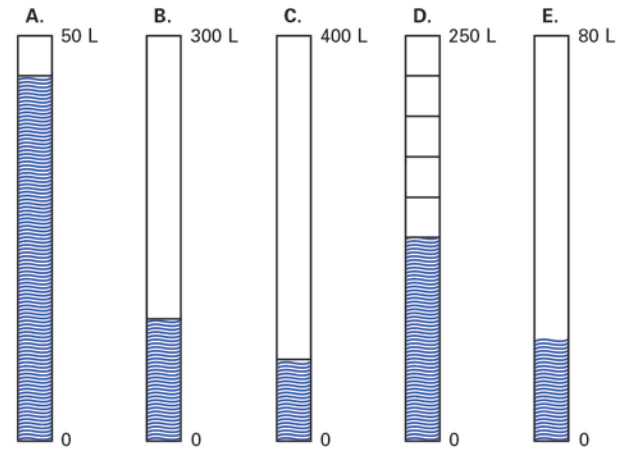
Notes

Point out that the label at the top of each bar represents the number of liters the tank holds.

15 Some students may have difficulty with this concept and will need to be re-directed to their answers to problem 13.

This week, Tim and Waya have to take care of watering all of the plots. They will connect a hose to one of the water tanks. They want to use the tank that has the most water.

12. Describe how Tim and Waya might determine which tank they will use.

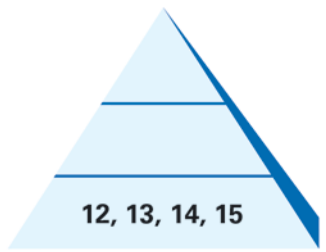


13. What part of each tank is filled? Write your answer as a fraction next to the shaded part of each tank on **Student Activity Sheet 5**.

14. How many liters of water are in each tank? Write your answer next to the shaded part of each tank.

15. **Reflect** Which tank would you suggest Tim and Waya use?

Assessment Pyramid



Use fractions for describing a part of a whole.

Reaching All Learners

Intervention

You may wish to make extra copies of **Student Activity Sheet 4** so that struggling students can cut, fold, and compare the bars.

English Language Learners

Have students put the bars in order from greatest to least and write each fraction below. Then they can write problems where they find the sums of two or more tanks.

Solutions and Samples

12. Answers may differ. Sample student responses:

- Tim and Waya can draw marks on the gauges for all tanks; they may use a separate strip of paper to do this. They can use the marks to find out the amount of water in each tank. They may first label each mark with a fraction.
- Tim and Waya can use the part that is shaded to find out how many of these parts are in one gauge. They can use this to find how much water is in the tank.

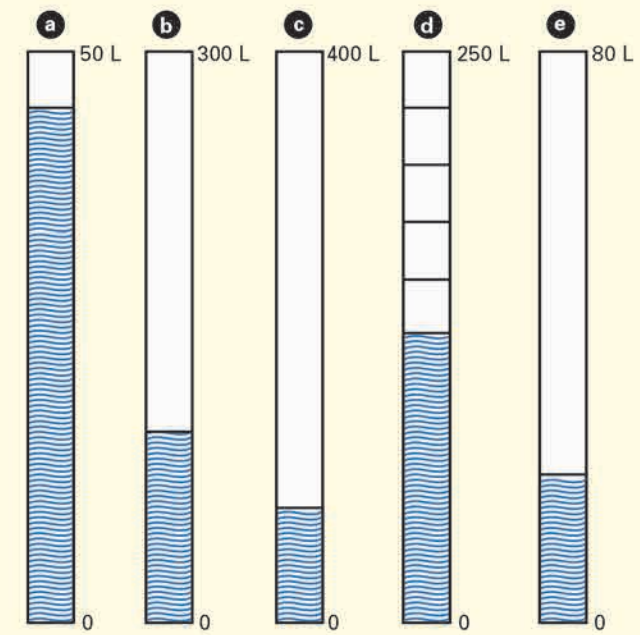
13. tank **A**: $\frac{9}{10}$ (Students can use the white part to "measure" on the gauge and make marks.)

tank **B**: $\frac{1}{3}$

tank **C**: $\frac{1}{5}$

tank **D**: $\frac{1}{2}$

tank **E**: $\frac{1}{4}$



14. See drawing in solution for problem 13.

tank **A**: each mark is 5 L, so $9 \times 5 = 45$, or $50 - 5 = 45$ L.

tank **B**: 100 L, one third of 300 L is 100 L.

tank **C**: 80 L, one fifth of 400 L is 80 L.

tank **D**: 125 L, half of 250 L is 125 L.

tank **E**: 20 L, one quarter of 80 L is 20 L.

15. tank **d**, because this one contains most water.

Hints and Comments

Materials

Student Activity Sheet 5 (one per student)

Overview

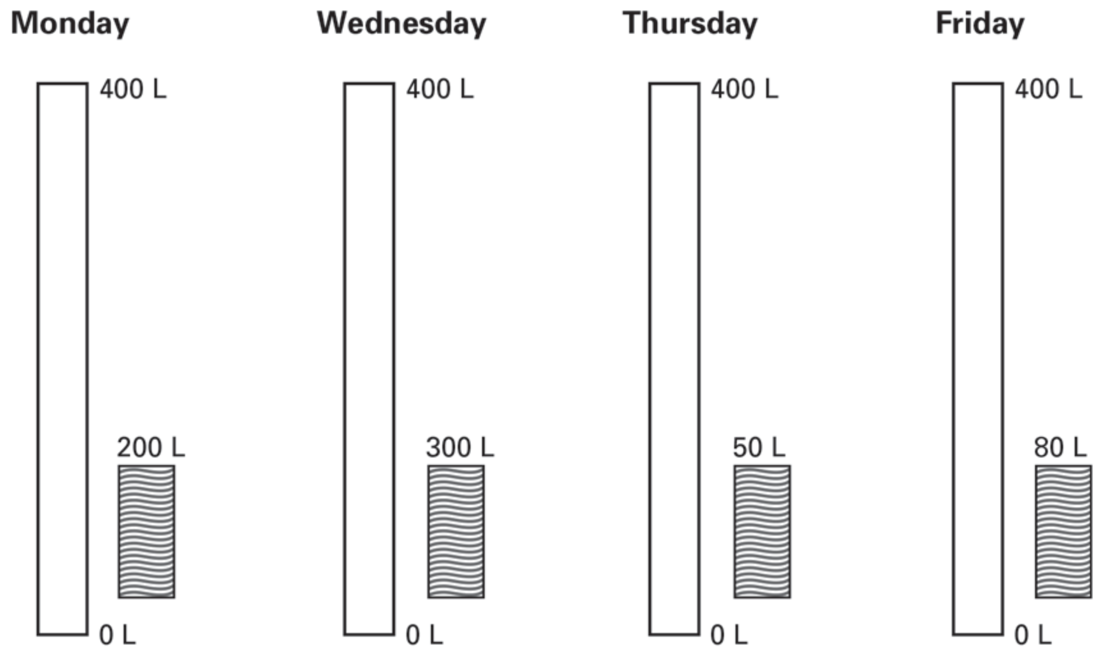
Given the capacity of each tank and the water level indicated by the gauges, students decide which tank out of five has the most water.

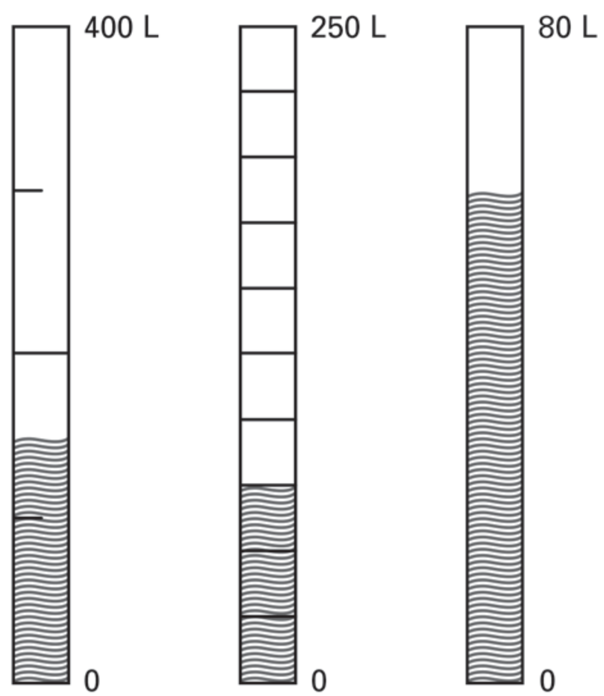
About the Mathematics

These problems again focus on making relative comparisons at an informal level. The concepts of absolute and relative comparisons are made explicit in the unit *Ratios and Rates.*

Comments About the Solutions

12. Accept all answers. Students revisit this concept in problem 13. Some students may think that the higher the gauge is shaded the more water there is in the tank. They may choose **A.** These students do not take into account the capacity of each tank.


Monday
Wednesday
Thursday
Friday
400 L
400 L
400 L
400 L
200 L
300 L
50 L
80 L
0 L
0 L
0 L
0 L


400 L
250 L
80 L
0
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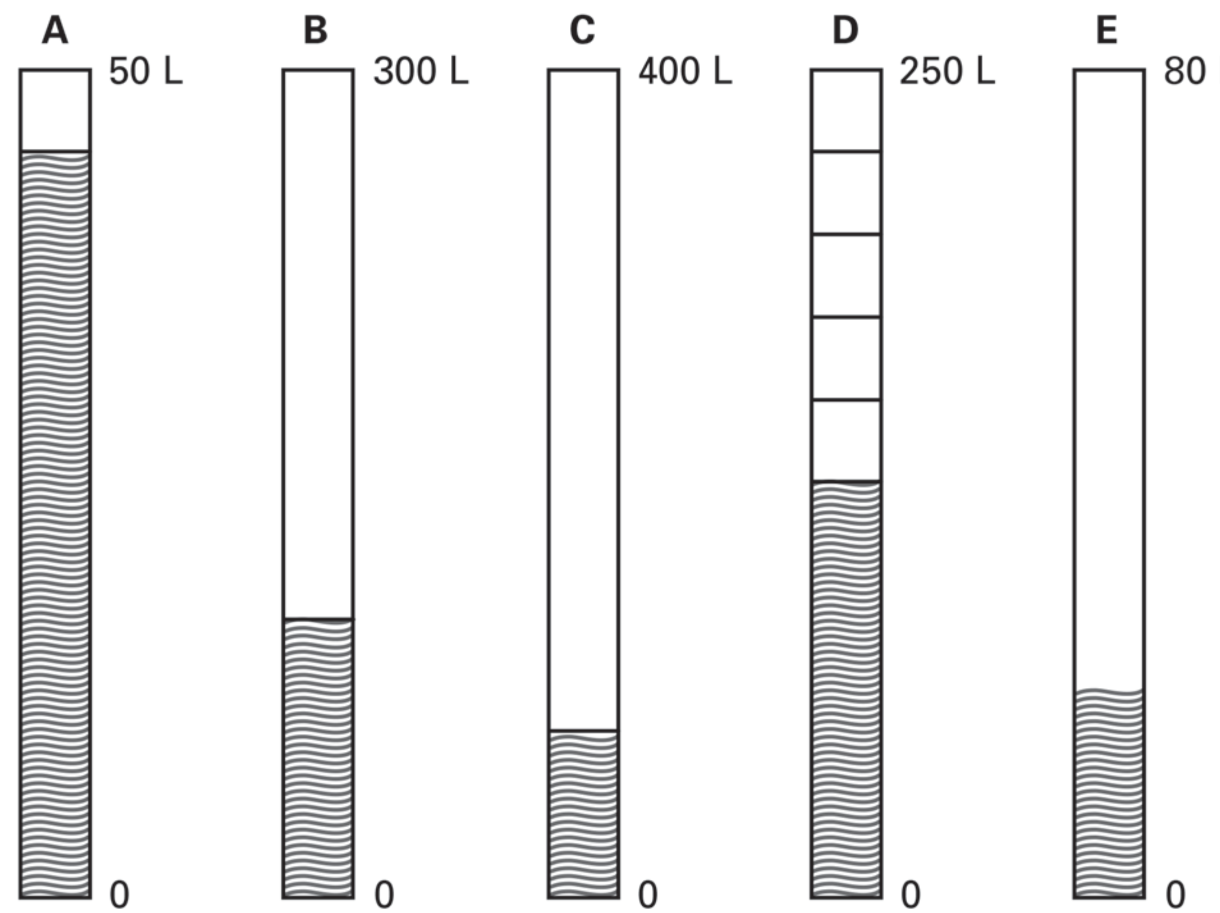

A
B
C
D
E
50 L
300 L
400 L
250 L
80 L
0
0
0
0
0

Notes

This page sets the context for the beginning of the section. Be sure that students understand that copy machines can make enlargements and reductions, but resist having a discussion of the meaning of the percentages given. Students will grapple with this in problem 1.

Enlarge or Reduce

Maritza, Laura, and Jamel are opening a new store called Roll On. To advertise the grand opening, Maritza and Jamel designed a flyer with a picture of an in-line skater.

Here is the picture that Maritza and Jamel want to use for the flyer.

They realized that the picture had to be reduced to fit on the flyer. Laura suggested that they use a photocopier to see what the reduced picture would look like. Jamel and Maritza agreed. They found a photocopier that could reduce originals to 25 percent and enlarge originals to 400 percent.

Reaching All Learners

English Language Learners

Be sure students understand what a flyer is since this is the context for the first problems.

Hints and Comments

Overview

Students apply and extend their knowledge of percent decrease and increase as they investigate a problem context involving the reduction and enlargement of flyers.

About the Mathematics

In this section, making reductions and enlargements using a photocopier helps to develop the concept of percents as operators.

Planning

Begin this section by asking students about their experiences using copy machines. Focus the discussion on machines that can reduce and enlarge. Some students may know that these machines use percents to indicate how much they enlarge or reduce. Show students that a 75% reduction produces a copy whose dimensions are 75% of the dimensions of the original.

Notes

2a Have students assume that the original dimensions are 10 cm × 15 cm.

3 Students should not be surprised that the result of two 50% reductions is not a 100% reduction. Once students get their result in part **c**, ask why this number makes sense.

3bc Encourage students to use arrow language.

4 Remind students to use ratio table strategies. Let them know that they do not have to complete the table in the order presented.

1. **Reflect** What does it mean to reduce to 25 percent and enlarge to 400 percent? Give examples to illustrate your explanation.

2. a. Suppose they reduce the picture to 50%. What will the new width and length be? Show your calculations.

b. Complete the arrow string to describe a reduction to 50%.

original length $\xrightarrow{\times \ldots}$ reduced length

The result of this reduction is still too large to fit on the flyer.

Maritza suggests, "Just take the reduced copy and reduce it again to 50%. Then we will see if that fits."

3. a. What are the width and length after two successive reductions to 50%?

b. Describe the calculation to make two reductions of 50%.

c. How can they get the same result, starting with the original and using just one reduction?

The group has gone to a lot of trouble to find the effect of a reduction. It would be a lot easier if the print shop had a chart that shows the measurements of an object after it is reduced.

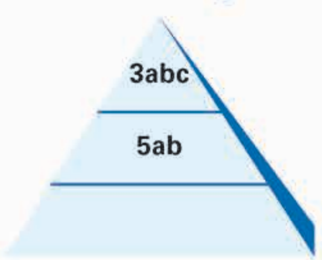
4. Copy and fill in the table below for making a reduction to 30%.

Original Length (in cm)	10	15	20	1	2	3	4	5
Length Reduced to 30%								

5. a. How can you use a calculator to find the effect of a reduction to 30%?

b. Use arrow language to describe this calculation.

Assessment Pyramid



Understand multiplicative decrease.

Use percents as operators.

Reaching All Learners

Intervention

In problem 2, be sure that students write either $\frac{1}{2}$ or 0.5. Some students have a tendency to write 2, thinking about dividing by two.

Accommodation

You can copy the ratio table from problem 4 so that students do not have to spend time drawing the table.

Solutions and Samples

1. Answers will vary. Sample response:
   - When a picture is reduced to 25%, the dimensions of the sides are $\frac{1}{4}$ of the original dimensions.

   When a picture is enlarged to 400%, the dimensions of the sides are 4 × the original dimensions.
   - If a picture measures 16 cm by 20 cm, it will be reduced to 4 cm by 5 cm. (25% of 16 cm is 4 cm and 25% of 20 cm is 5 cm).

   If a picture measures 2 cm by 3 cm, it will be enlarged to 8 cm by 12 cm (400% is four times).

2. a. This answer assumes that the picture dimensions are 10 cm by 15 cm. Width = 5 cm, length = 7.5 cm.

   b. original length $\xrightarrow{\times \frac{1}{2}}$ reduced length

3. a. Width = 2.5 cm, length = 3.75 cm.

   b. Answers may vary.
   - divide by 2 and then divide by 2 again
   - half and then half again
   - divide by 4
   - multiply by 0.25 (half of half is one-fourth which is 0.25)
   - original length $\xrightarrow{\times \frac{1}{2}}$ ___ $\xrightarrow{\times \frac{1}{2}}$ reduced length

   c. By reducing to 25%

4.

Original Length (in cm)	10	15	20	1	2	3	4	5
Length Reduced to 30%	3	4.5	6	0.3	0.6	0.9	1.2	1.5

5. a. Multiply the original length and width by 0.3.

   b. original length $\xrightarrow{\times 0.3}$ reduced length

Hints and Comments

Materials

centimeter ruler (one per pair of students)

Overview

Students explain what a photocopy machine actually does when it reduces a picture to 25% and enlarges it to 400% of the original size. Students use arrow language to describe reductions. They use arrow language and a calculator to find the dimensions of both first and second reductions.

About the Mathematics

The use of arrow language will further develop the idea of percents as an operator. Students may remember from the unit *Expressions and Formulas* how to work with arrow language.

Some students may confuse size with dimensions. When the picture is reduced to 80%, for example, the dimensions are reduced to 80%. The area (or size), however, is reduced to 64% of the original.

The concepts of reduction, enlargement, ratio, and scale changes will be revisited in the unit *Ratios and Rates.*

Planning

Students may work in small groups on problems 1–5. A class discussion should emphasize students' solutions and strategies for problems 3b and 4.

Comments About the Solutions

1. You may want to make a distinction between "reducing by 80%" and "reducing to 80%." For example, a picture 10 cm by 10 cm reduced "to" 80% means the length and width are each reduced by 20%, or they are 80% of their original length. The picture becomes 8 cm by 8 cm.

   When the 10 cm by 10 cm picture is reduced "by" 80%, it means that the dimensions are reduced by 80% (or reduced to 20%), and the length of each side will become 2 cm.

Extension

Problem 3 provides an opportunity to discuss the relationship between the length and area of the original picture and the length and area of the reduced picture. Some students may notice that the reduced picture has dimensions half of the original picture, but it fits four times on the original picture. Therefore, the area is $\frac{1}{4}$ of the area of the original picture.

Notes

8 This problem is difficult. Part **a** asks students if they think it is possible and why they think that without their having to prove it. Part **b** asks for the numbers to make it work.

9 This should be a review of the previous section for students. Make sure students are clear that Maritza is calculating the discount, not the sale price.

The group wants to make a poster using the original picture. This time the picture has to be enlarged.

6. Find the dimensions of a picture 10 centimeters (cm) by 15 cm enlarged to 200%. Show your calculations.

The result is too small for the poster, so they decide to enlarge the original picture to 250%.

7. a. Find the dimensions of the picture (10 cm by 15 cm) enlarged to 250%. Show your calculations.

b. Use arrow language to describe this calculation.

Suppose you want to make an enlargement to 200%. The photocopier you are using enlarges to only 150%.

8. a. Will two enlargements to 150% give the desired result? Explain.

b. Find two enlargements that can be used with this photocopier to produce a final enlargement as close as possible to 200%. Copy the arrow string to describe your result.

original length $\xrightarrow{\times \ldots}$ .... $\xrightarrow{\times \ldots}$ enlarged length

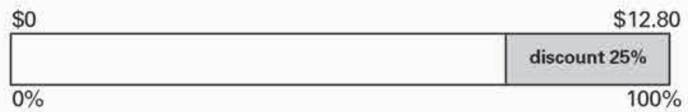
Discount

Maritza and Jamel went to the Office Supply Store to buy a frame for the poster. There were several frames for sale. Maritza liked the one shown on the left.

9. a. What is the discount in dollars?

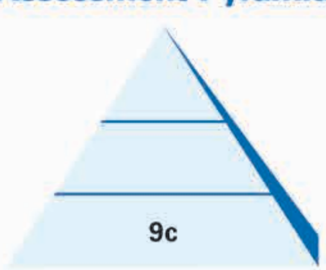
b. Maritza calculated the discount with one multiplication: 0.25 × $12.80.

Explain why this is correct. The percent bar can be helpful for finding an explanation.



c. Calculate the sale price for this frame.

Assessment Pyramid



Calculate sale and discount price.

Reaching All Learners

Extension

Students could be challenged in problem 7 to figure out how to set the machine if they were going to use the 200% enlargement to get the 250% enlargement.

Intervention

For students who are having difficulty with problem 8b, ask, *What has to be true about the two numbers you are looking for?* (The product of the numbers must be two.)

Solutions and Samples

6. The dimensions are 20 cm by 30 cm.
Sample calculation:
10 × 2 = 20 cm and 15 × 2 = 30 cm.

7. a. The dimensions are 25 cm by 37.5 cm.
Sample calculation:
10 × 2.5 = 25 cm and
15 × 2.5 = 37.5 cm.

b. original length $\xrightarrow{\times 2.50}$ enlarged length

8. a. No.

Sample explanations:

- original length $\xrightarrow{\times 1.5}$ ___ $\xrightarrow{\times 1.5}$ enlarged length is the same as
  original length $\xrightarrow{\times 2.25}$ enlarged length
- If I have a picture with dimensions of 4 cm by 4 cm and I enlarge it 150%, then the new dimensions are 6 cm by 6 cm. If I enlarge this to 150%, then the new dimensions are 9 cm by 9 cm, and this is more than two times the original dimensions.
- If you calculate *number* × 1.5 ×1.5, then it is the same as *number* × 2.25, and this is more than × 2.

b. Answers may vary.

Sample answers:

- Two enlargements to 140%, gives

original length $\xrightarrow{\times 1.4}$ ___ $\xrightarrow{\times 1.4}$ enlarged length, or original length $\xrightarrow{\times 1.96}$ enlarged length (1.4 × 1.4 = 1.96)

- I first used an enlargement of 150%. Now I have to find a number so that 1.5 × ? = 2

I calculated 2 ÷ 1.5 which is 1.33, so the second enlargement is 133%.

9. a. $3.20

b. 25% of something is the same as $\frac{1}{4}$ part, or times $\frac{1}{4}$, and this is same as times 0.25.

c. The sales price is $9.60.

Hints and Comments

Materials

calculators (one per student)

Overview

Students calculate the dimensions of the picture enlarged to 200% and to 250%. They also use arrow language to describe the 250% enlargement. They find a first and second enlargement that will have the same result as one enlargement of 200%.

Students reinforce their understanding of the use of a percent as operator in the context of discount.

About the Mathematics

Students may remember from the unit *Expressions and Formulas* how two arrows can be shortened to one arrow. (Refer to the solutions to problem 8 in the solution column.)

Planning

You may have students work in small groups on problems 6-8. Problem 9 can be assigned as homework or used as Informal Assessment.

Comments About the Solutions

8. Since there are no picture dimensions given, some students may not know how to solve this problem. You may give the hint that they are allowed to use a picture with dimensions they make up by themselves, for example 4 cm by 4 cm.

8. b. The two enlargements do not have to be the same percentage.

An accurate answer can be found by using the square root of 2, which gives two enlargements of 141%.

Students can use a strategy of trial and error, but they can also try to develop a more straightforward strategy.

9. Students' explanations will show how they developed their number sense and their ability of using a percentage as an operator.

Extension

Ask students, *How would you enlarge a picture to 150% using a photocopy machine that only enlarges to 120%?* Students will notice that you have to enlarge it more then two times.

Possible solutions:
114%, 114%, 114% results in 148% of the original;
120%, 120%, 104% results in almost 150%.

Enlarge or Reduce

Maritza, Laura, and Jamel are opening a new store called Roll On. To advertise the grand opening, Maritza and Jamel designed a flyer with a picture of an in-line skater.

Here is the picture that Maritza and Jamel want to use for the flyer.

They realized that the picture had to be reduced to fit on the flyer. Laura suggested that they use a photocopier to see what the reduced picture would look like. Jamel and Maritza agreed. They found a photocopier that could reduce originals to 25 percent and enlarge originals to 400 percent.

1. **Reflect** What does it mean to reduce to 25 percent and enlarge to 400 percent? Give examples to illustrate your explanation.

2. **a.** Suppose they reduce the picture to 50%. What will the new width and length be? Show your calculations.

   b. Complete the arrow string to describe a reduction to 50%.

original length $\xrightarrow{\times \ldots}$ reduced length

The result of this reduction is still too large to fit on the flyer.

Maritza suggests, "Just take the reduced copy and reduce it again to 50%. Then we will see if that fits."

3. **a.** What are the width and length after two successive reductions to 50%?

   b. Describe the calculation to make two reductions of 50%.

   c. How can they get the same result, starting with the original and using just one reduction?

The group has gone to a lot of trouble to find the effect of a reduction. It would be a lot easier if the print shop had a chart that shows the measurements of an object after it is reduced.

4. Copy and fill in the table below for making a reduction to 30%.

Original Length (in cm)	10	15	20	1	2	3	4	5
Length Reduced to 30%								

5. **a.** How can you use a calculator to find the effect of a reduction to 30%?

   b. Use arrow language to describe this calculation.

Notes

Introduce this section by asking if students know the local sales tax policy. Depending on students' previous experiences, you may need to explain that states have different sales taxes.

9a If students struggle with this question, begin by asking them what each of the numbers in the equation means. Ask, "What does the 50 stand for?" and "What does the 2 mean?"

T-Shirts and Jeans

Robin and Jamie went shopping at an outlet store in a neighboring state because that state has no sales tax on clothes. The store policy is that all T-shirts have the same price, all polo shirts have the same price, all jeans have the same price, and all shorts have the same price. Both Robin and Jamie bought T-shirts and jeans on this shopping trip.

Their friend Lisette wants to know the price of one T-shirt and also the price of one pair of jeans, but Robin and Jamie only remember how many of each item they bought and how much they paid in total.

Robin bought 2 T-shirts and 2 pairs of jeans and paid $50.

Jamie bought 3 T-shirts and 1 pair of jeans and paid $45.

8. Based on this information, is it possible to find out how much one T-shirt costs and how much one pair of jeans costs? Explain your answer.

The equation that represents Robin's purchases is $2x + 2y = 50$.

9. a. What do the x and the y represent in this equation?

b. List three coordinate pairs that would satisfy this equation. Remember to keep in mind what the variables represent when finding your coordinate pairs.

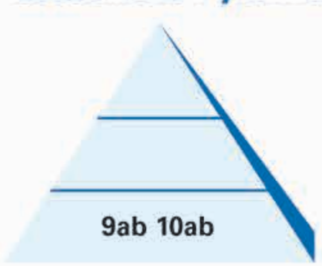
A similar equation can be written to represent Jamie's purchases.

10. a. Write an equation to represent Jamie's purchases.

b. List three coordinate pairs that would satisfy this equation.

Remember to keep in mind the context of the problem when finding your coordinate pairs.

Assessment Pyramid



Translate problems involving two unknowns into an algebraic equation.

Reaching All Learners

Intervention

Students can check the validity of their solutions for the problems on this page by substituting numbers in the coordinate pairs back into the equation. Also ask students to determine whether their coordinate pairs would or would not make sense in the problem. For example, $(\frac{44}{3},1)$ fits Jamie's equation but $\$\frac{44}{3}$ is not a likely price for a T-shirt.

Solutions and Samples

8. Yes it is possible by writing a linear equation that represents Robin's purchase and another equation that represents Jamie's purchase, drawing the graphs of the lines, and finding the intersection point.

9. a. The x represents the number of T-shirts and the y represents the number of pairs of jeans.

b. Answers will vary. Possible solutions: (5,20), (7.5,17.5), (10,15)

10. a. Jamie bought 3 T-shirts ($3x$) and (+) 1 pair of jeans (y) and paid (=) $45; $3x + y = 45$

b. Answers will vary. Possible solutions: (5,30), (7.5,22.5), (10,15)

Hints and Comments

Overview

Students determine the price of a T-shirt and the price of a pair of jeans by translating a problem context into two linear equations.

About the Mathematics

This section allows students to practice the skills they have learned in the previous pages, including translating a problem into algebraic equations and finding coordinate pairs that fit into these equations. On the next page, students will graph these algebraic equations and find the point of intersection, thereby finding the solution to the system of equations.

Comments About the Solutions

9. a. Some students may reverse the meaning of the variables. If they do, then their equation for Jamie's purchases will be different. Stress that one way is not better than the other, but that they must be consistent in the use of their variables. Some students might be more comfortable using the variable t for the cost of one T-shirt and the variable j for the cost of one pair of jeans. Then the equations become $2t + 2j = 50$ and $3t + 1j = 45$. The decision of which variable to assign to which axis still needs to be made. This can be a class decision or an individual one. Be prepared to guide students to compare results when the axes are labeled differently.

Notes

11a Have a discussion about why **Student Activity Sheet 8** looks different than **Student Activity Sheet 7**. Why are there different numbers used for the scales?

11b Ask students to explain if the line should continue to extend into the quadrants where x or y are negative numbers.

16 Note that this question is optional and can be omitted to save time if students can explain in question 15 that the lines will never intersect.

Encourage students to identify which variable represents which item. It is a good habit to write a formal explanation: Let x = the cost of one pair of shorts and let y = the cost of one polo shirt. Students can write the variables the other way as well as long as they are consistent.

11. **a.** Use **Student Activity Sheet 8** to graph the points you listed in problem 9.

b. Connect the points to draw a straight line.

c. Label the line Robin.

12. **a.** Use the same **Student Activity Sheet** to graph the points you listed in problem 10.

b. Connect the points to draw a straight line.

c. Label the line Jamie.

The lines have a point of intersection.

13. **a.** What are the coordinates of the point of intersection?

b. What is the meaning of the point of intersection?

c. What is the price of a T-shirt and what is the price of a pair of jeans in the outlet store?

Shorts and Polo Shirts

Hector says he bought 1 pair of shorts and 1 polo shirt and paid $22.50.

Charlie says he bought 1 pair of shorts and 1 polo shirt and paid $25.

14. Explain why you cannot find the price of a pair of shorts and a polo shirt based on the information above.

15. **Reflect** Without graphing the lines that represent Hector's and Charlie's purchases, do you think there is a point of intersection?

16. **a.** Write one equation to represent Hector's purchase and one equation to represent Charlie's purchase.

b. Graph the equations on **Student Activity Sheet 9**.

c. Is there a point of intersection?

Reaching All Learners

Intervention

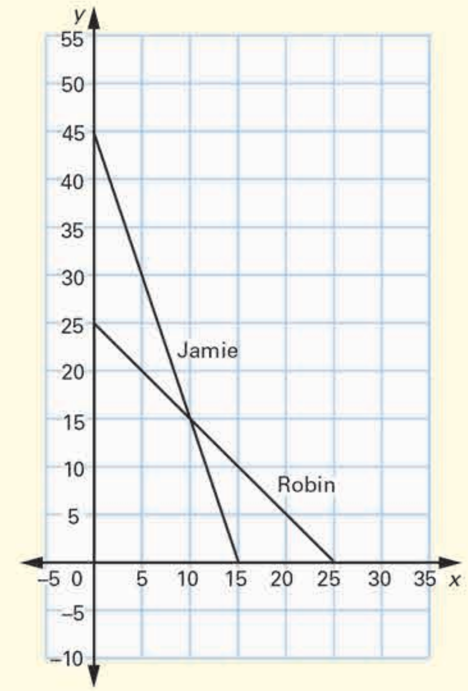
This might be a good time to point out that the purpose of drawing a straight line is to find the intersection point, but a straight line is actually representing the possible prices. Not all points on the line would be a possible solution to Jamie's scenario, but by drawing a connected line we are including all solutions. Have students find points on the line that do not make sense in the context of the problem.

Writing Opportunity

Problems 14, 15, and 16 can be framed as a persuasive report or essay that includes a graph as a visual component. Use problem 14 as the writing prompt and problems 15 and 16 as two components that must be included in the report.

Solutions and Samples

11 and **12.**



13. a. (10,15)

b. The intersection point is the point that gives a value for x and a value for y that satisfies both equations. It is the point that tells you the prices of a T-shirt and a pair of jeans.

c. A T-shirts costs $10 and a pair of jeans costs $15.

14. Explanations will vary. Sample explanation:

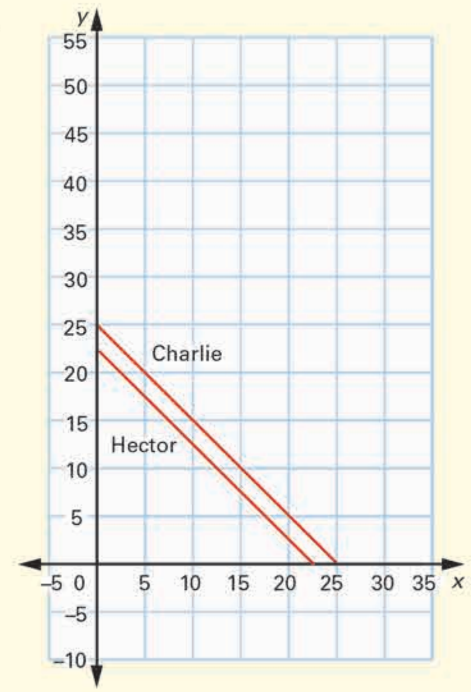
- You can't find the prices since they both bought the same items and paid different prices.

15. Responses will vary. Sample responses:

- I do not think there is a point of intersection since they bought the same items but paid different prices, so one person was charged more for one item or both items.
- I think there is a point of intersection since all lines intersect.

16. a. Hector: $x + y = 22.50$; Charlie: $x + y = 25$

b.



c. There is no point of intersection since the lines are parallel. This corresponds with the information in the problem meaning there is no point that satisfies both equations.

Hints and Comments

Materials

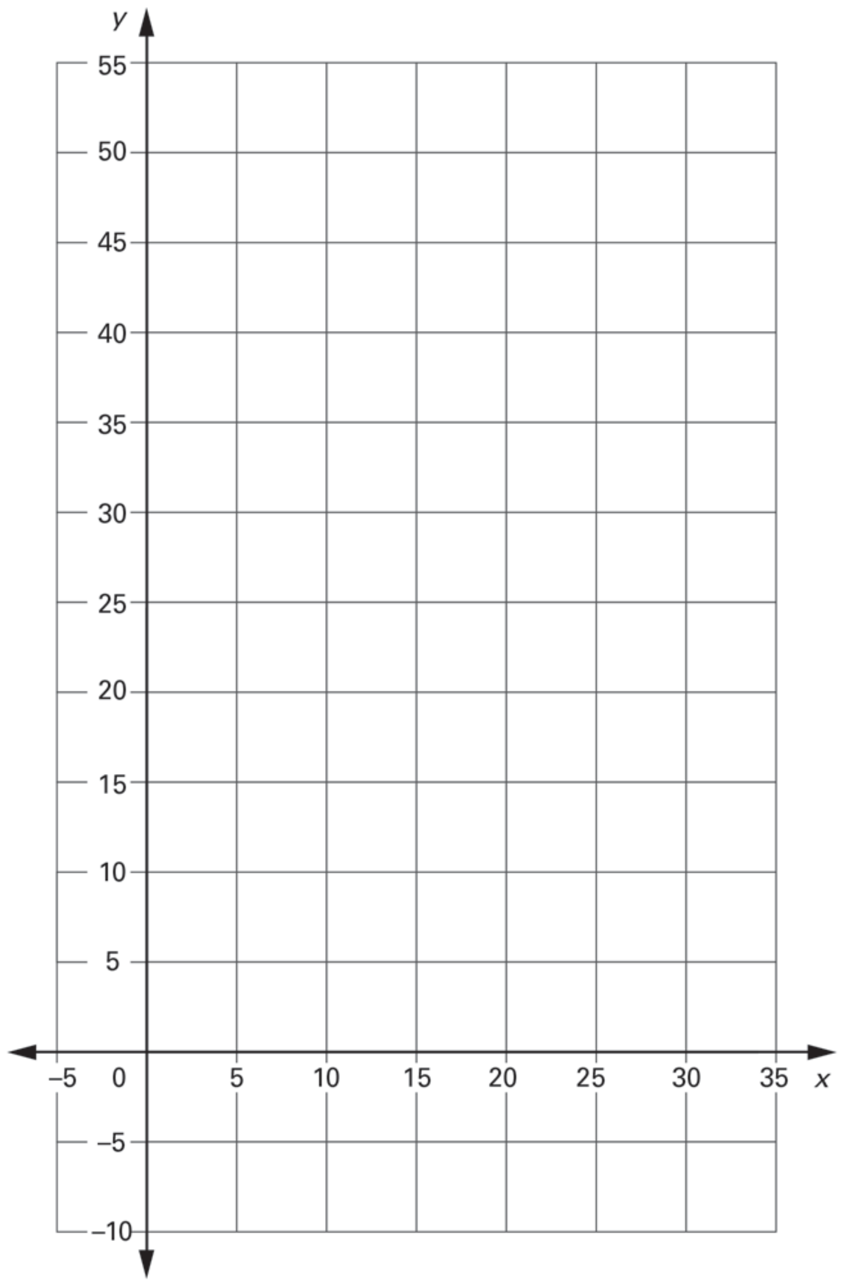
Student Activity Sheets 8 and **9**

Overview

Students determine the price of a T-shirt and the price of a pair of jeans in this context by graphing two linear equations and finding their point of intersection. They evaluate other equations that have no point of intersection.

About the Mathematics

A system of equations works only if the values of the variables do not vary from one equation to the other. In this case, Hector and Charlie both bought the same items but at different prices, which implies that the equations do not have consistent variables and do not have a common solution for each variable. This is shown by the graph of two parallel lines. Another pair of equations that would not yield a solution would be a pair of equations that overlap. For example, $x + y = 25$ and $2x + 2y = 50$ have the same graph. Although the variables are consistent in this case, the equations are equivalent to each other and do not provide a solution.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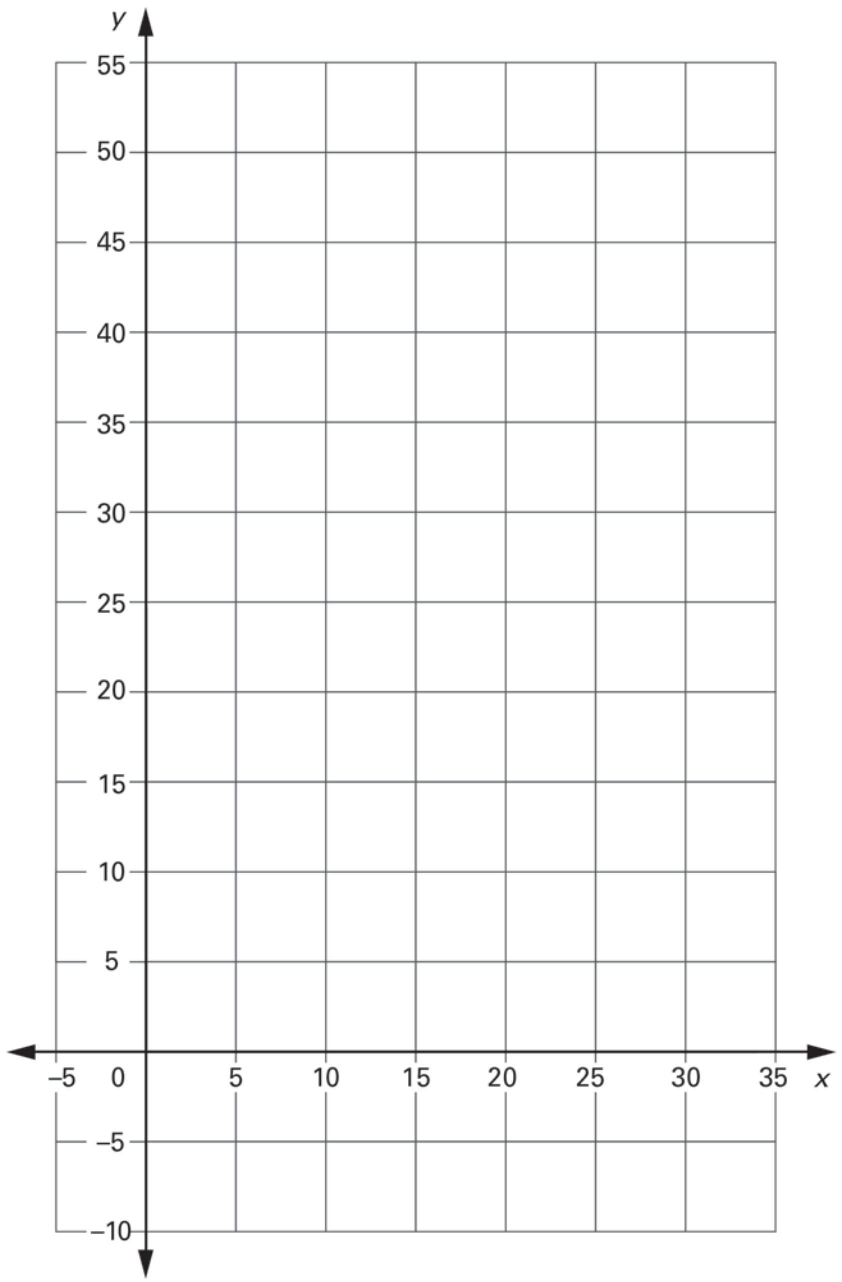
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Notes

5 Students use the eight given points to draw a graph. The graph should not have sharp zigzag edges, since the water level increases and decreases gradually. Students' graphs will vary in precision and neatness.

5 Remind students that 12 A.M. is midnight and 12 P.M. is noon.

Golden Gate Bridge

In different parts of the world, the levels of high and low tides vary. The amount of time between the tides may also vary. Here is a tide schedule for the area near the Golden Gate Bridge in San Francisco, California.

5. Use the information in the table to sketch a graph of the water levels near the Golden Gate Bridge for these three days. Use **Student Activity Sheet 12** for your graph.

Date	Low	High
Aug. 7	2:00 A.M./12 cm 1:24 P.M./94 cm	9:20 A.M./131 cm 7:47 P.M./189 cm
Aug. 8	2:59 A.M./6 cm 2:33 P.M./94 cm	10:18 A.M./137 cm 8:42 P.M./189 cm
Aug. 9	3:49 A.M./0 cm 3:29 P.M./91 cm	11:04 A.M./143 cm 9:34 P.M./186 cm

6. Describe how the water level changed.

7. Compare your graph to the graphs on the previous page. What similarities and differences do you notice?

Reaching All Learners

Advanced Learners

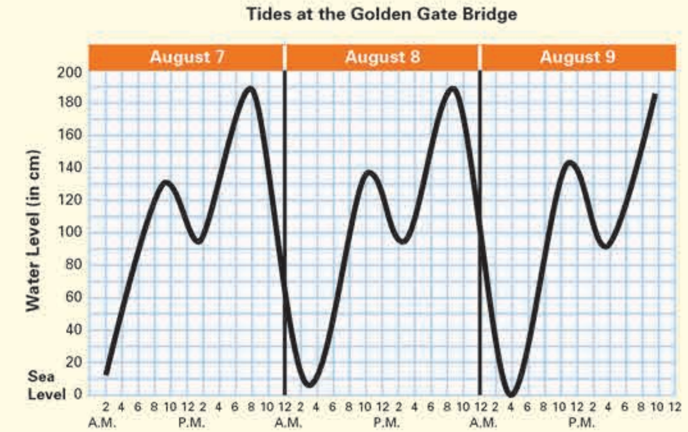
If you live near bodies of water that have tidal fluctuations, you can obtain local times for high and low tides in the local newspaper. Students could then use this information to draw additional tidal graphs.

Writing Opportunity

Make available a sample graph of fluctuation in tides from the Internet or some other resource. Have students write a story to go along with the graph. Also encourage students to make a table from the graph that indicates the highs and lows for each day, in a format similar to the table shown in problem 5.

Solutions and Samples

5. Sample graph:



6. Answers will vary, and some descriptions may be more detailed than others. For example, students may include extensive information about times and heights. Sample responses:

   On the first day, the tide rose for about seven hours, and at 9:20 A.M. it was high tide. Then it fell for about four hours. Then the tide rose again for about six hours. When high tide was reached at about 8:00 P.M., the level was higher than the high tide level in the morning. Then for the rest of the day, the tide fell. On the second and third days, the changes in the water level show the same pattern as on the first day.

   There are two high and two low tides each day. It seems like the low tides are getting lower and the high tides are getting higher.

7. Answers will vary. Sample responses:

   All the graphs show that there are two high tides and two low tides for each day. The difference is that in San Francisco, the water levels for the two high tides in a single day differ quite a bit. The same thing is true for water levels for the low tides. In the graphs of problem 5, the water levels for the high tides on a single day are almost the same.

   The graph of the tides in San Francisco varies more than the graph of the coastal tide flats.

   The water level in the tide flats changes from below sea level to above sea level; the water level in San Francisco is always at or above sea level.

Hints and Comments

Materials

Student Activity Sheet 12 (one per student); graph paper (one sheet per student)

Overview

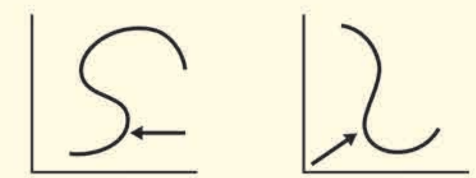
Students use the data in a table about the times and levels of low tide and high tide to sketch a graph of the water level at the Golden Gate Bridge over a period of three days. Then they compare this graph with those on the previous page.

Planning

Students may work in pairs on problems 5–7. You may want students to finish these problems as homework.

Comments About the Solutions

Observe that no tidal graph shows curves like this.



(These graphs show three different water levels at the same time, which is impossible.)

To help students get started, discuss how to plot points for the first two hours of the day on August 7.

6. Students' answers should include a description of the two (relative) high tides and low tides on one day, and the repeating pattern.

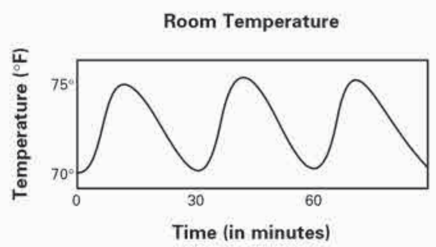
7. Differences in water levels of high tides and low tides at the Golden Gate Bridge and the Dutch Shallows are due to the differences in the shapes of the coastlines near the two bodies of water.

Notes

By this point in the unit, students should be able to think about periodicity in terms of both the real-world context and the graph.

9a and **b** This is the first problem in which students must identify the period and cycle in a periodic graph. Point out that a graph's cycle can be colored at different places on the graph curve. Once one cycle is colored, it makes it easy to identify the period of the graph, the time interval along the horizontal axis that it takes to complete one cycle.

The Air Conditioner



Suppose the graph on the left shows the temperature changes in an air-conditioned room.

8. Describe what is happening in the graph. Why do you think this is happening?

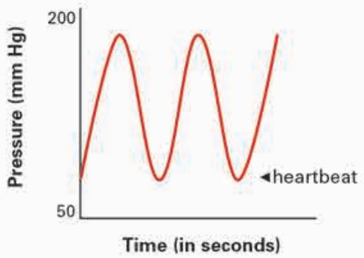
The graphs you have seen in this section have one thing in common: They have a shape that repeats. A repeating graph is called a **periodic graph**. The amount of time it takes for a periodic graph to repeat is called a **period** of the graph. The portion of the graph that repeats is called a **cycle**.

9. a. How long (in minutes) is a period in the above graph?
   b. On **Student Activity Sheet 13**, color one cycle on the graph.

Blood Pressure

Your heart pumps blood throughout your system of arteries. When doctors measure blood pressure, they usually measure the pressure of the blood in the artery of the upper arm.

Your blood pressure is not constant. The graph on the right shows how blood pressure may change over time.

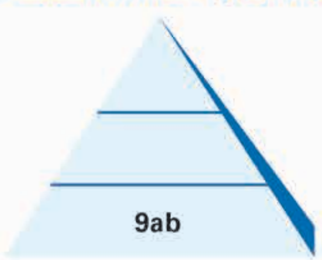


10. What can you tell about blood pressure just before a heartbeat?

11. What happens to blood pressure after a heartbeat?

12. Is this graph periodic? Explain your answer.

Assessment Pyramid



Identify characteristics of periodic graphs.

Reaching All Learners

Vocabulary Building

If students are having difficulty, you might draw a different example of a periodic graph and discuss the concepts of period and cycle with students.

Extension

You might discuss how the graph would change if the person had been running and the heartbeat was twice as fast.

Solutions and Samples

8. Answers will vary. Some students may comment that the temperature rises when the air conditioner is off and falls when it is on.

9. a. The period in the graph shown is 30 minutes.

b. Answers will vary, depending on where a student starts to color a cycle. A cycle is correctly shown if it covers the time period that corresponds to students' answers for problem 9a.

10. Answers will vary. Sample response:

The blood pressure is at its lowest point just before the heart beats.

11. Answers will vary. Sample response:

After a heartbeat, the blood pressure rises rapidly, then rises more slowly, and finally stops rising. The pressure then begins to fall, slowly at first, then more rapidly, then more slowly again until it reaches its lowest value.

12. Yes, this is a periodic graph because the cycle repeats.

Hints and Comments

Materials

Student Activity Sheet 13 (one per student)

Overview

Students interpret a graph that shows a repeating pattern of temperature changes in an air-conditioned room. Students are introduced to a new context addressing a cyclic process, blood pressure. They investigate a graph that shows how blood pressure can change over time.

About the Mathematics

The concepts of a periodic graph, a period, and a cycle are made explicit on this page. Although these are new mathematical concepts, students may already have some understanding of periodic events, since repeating patterns are frequently found in science and nature. For example, sound and light waves show repeating patterns that can be illustrated using a periodic graph. High and low tides, phases of the moon, as well as the human heartbeat are also characterized by repeating patterns that make periodic graphs. Note that sometimes a phenomenon approximates a regular pattern but is not exactly periodic. In such cases, the periodic function serves as a model.

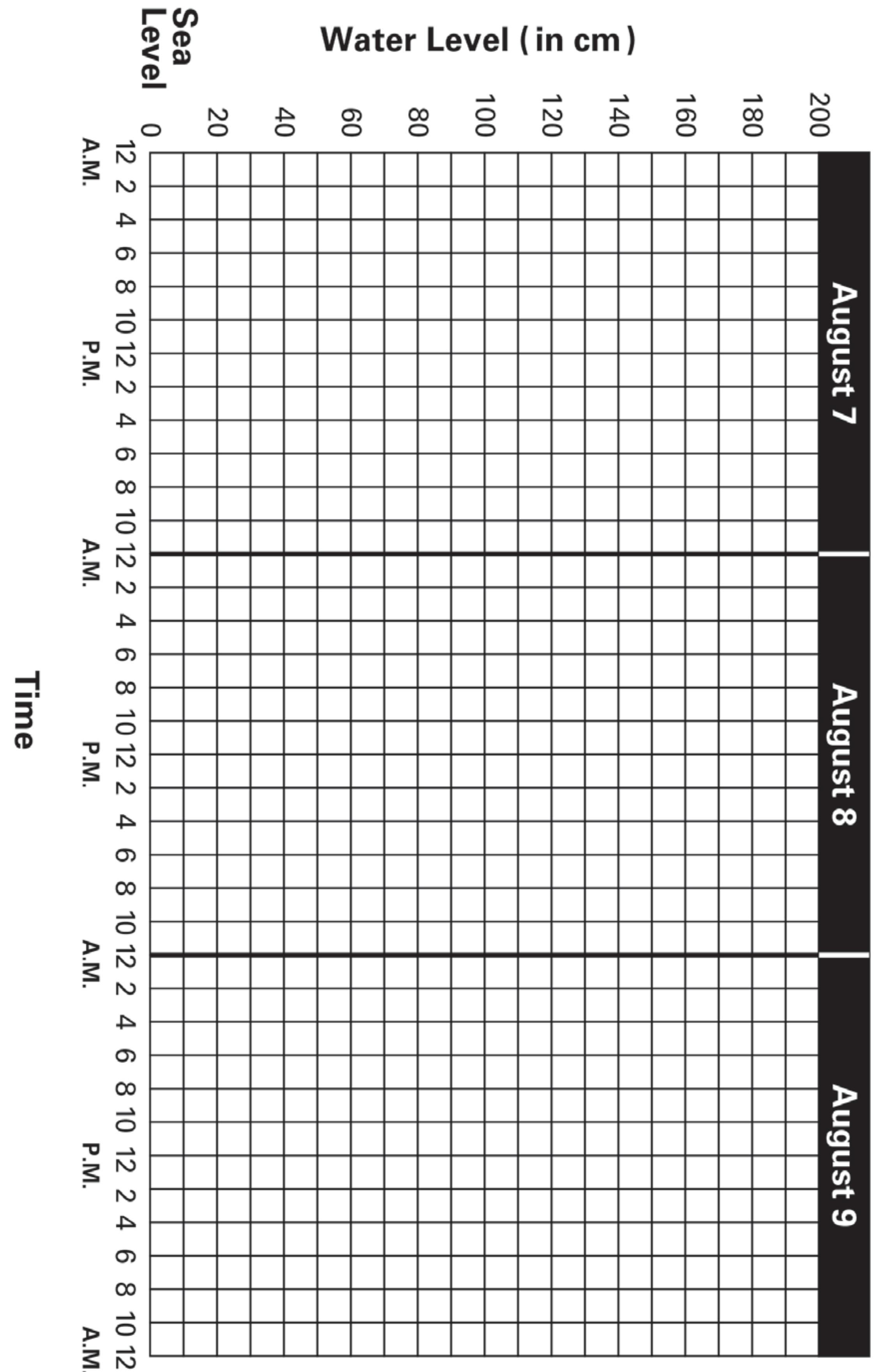
Planning

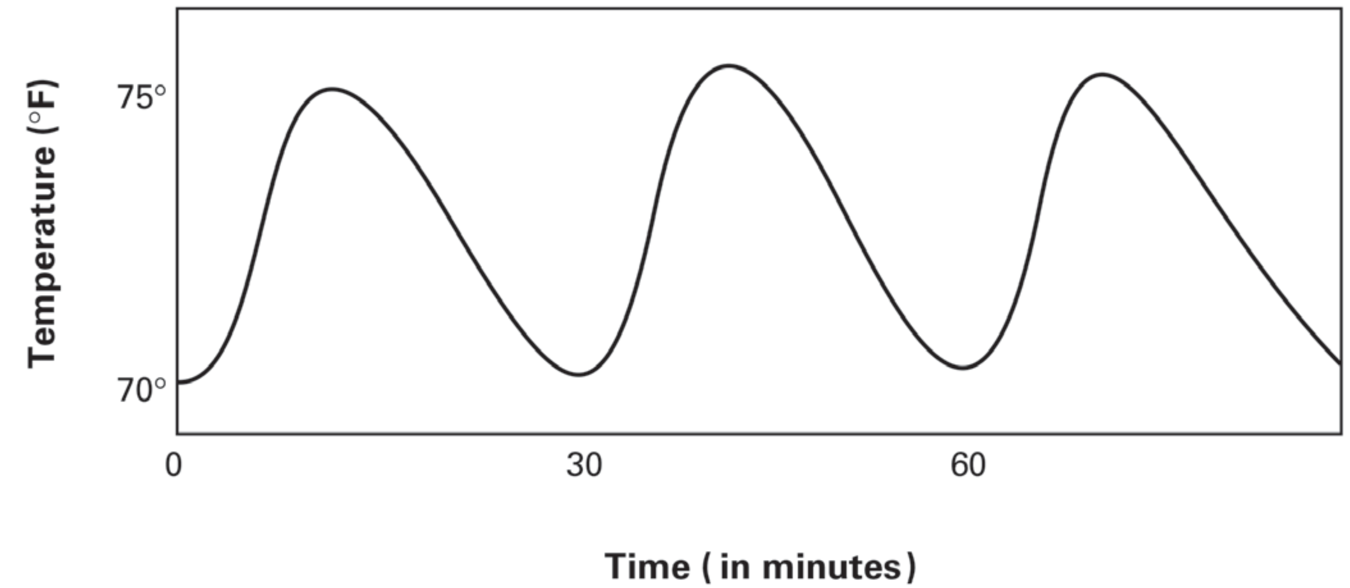
You may want students to work individually on problems 10–12.

Extension

You may have students work in pairs or small groups on the following activity.

1. Use a reference book or experiment to find the following information:
   - How many times do you inhale and exhale in one minute?
   - How much air do you inhale when you breathe normally?
   - What is the volume of your lungs?
2. Make a graph that shows the volume of your lungs over a time period of 15 seconds.
3. If your graph is periodic, how long is a period? If it is not periodic, explain why.





9. **b.** Color one cycle on the graph.

Mathematics in Context Modules

GRADE 6 COMMON CORE SEQUENCE

MODELS YOU CAN COUNT ON
Math models for comparing and computing ratios using fractions, decimals, and percents

EXPRESSIONS AND FORMULAS
Application of algebraic expressions using properties of operations and order of operations

FRACTION TIMES
Operations with fractions including division; ordering fractions and determining equivalents

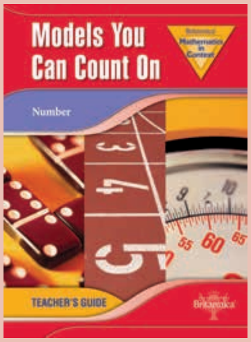
OPERATIONS
Integer operations, modeling on number lines, order of operations; coordinate systems

COMPARING QUANTITIES
Reason about and find informal solutions of systems of equations; concept of variable

FACTS AND FACTORS
Finding common factors, place value, number theory; exponential notation

REALLOTMENT
Area, perimeter, surface area, and volume

DEALING WITH DATA
Display and describe data distribution; measures of center; sampling techniques



GRADE 7 COMMON CORE SEQUENCE

MORE OR LESS
Ratios and percent problems using tape diagrams; proportional relationships by equations

RATIOS AND RATES
Ratios, unit rates, and percents as linear functions
and relationships; scale drawings and factors

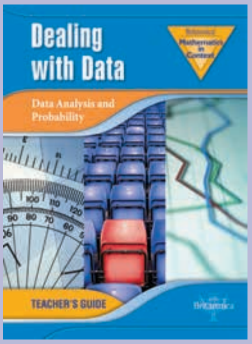
GO RATIONAL! (PDF ONLY)
Extend operations with fractions to calculate all four operations with rational numbers

BUILDING FORMULAS
Patterns that lead to formulas; equivalent expressions; operations with rational numbers

TRIANGLES AND BEYOND
Draw, construct and describe geometrical figures; transformations, congruence

PACKAGES AND POLYGONS
Two-dimensional representations of three-dimensional prisms; volume

SECOND CHANCE
Determining chance; theoretical and experimental probabilities; sample and compare



GRADE 8 COMMON CORE SEQUENCE

REVISITING NUMBERS
Integer exponents, scientific notation; operations with rational numbers; volume of 3D shapes

UPS AND DOWNS
Evaluate and compare functions using tables, graphs, and formulas; exponential functions

IT'S ALL THE SAME
Congruence and similarity; symmetry; Pythagorean Theorem

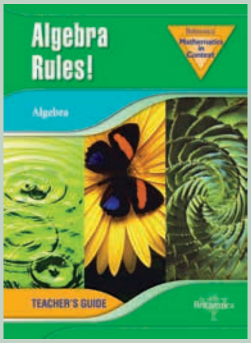
GRAPHING EQUATIONS
Understand slope of a line; solve linear equations; find point of intersection of linear equations

PATTERNS AND FIGURES
Patterns; construct a function; create and solve equations in one variable

ALGEBRA RULES!
Connect linear functions with models, graphs, and context; factor linear equations

INSIGHTS INTO DATA
Scatter plots; representations of data; conclusions based on data collection

GREAT PREDICTIONS
Construct and interpret two-way tables; describe association between variables



Additional Titles Available

PICTURING NUMBERS
Mean, median, and mode; displaying and interpreting data

FIGURING ALL THE ANGLES
Angle measure; rectangular grids; direction

TAKE A CHANCE
Independent events; counting strategies; multiple representations of simple probabilities

MADE TO MEASURE
Estimating and measuring length, area, and volume

LOOKING AT AN ANGLE
Right triangle relationships; three-dimensional views of two-dimensional drawings

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